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CONTENTS

FIGURES

TABLES

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I. INTRODUCTION

1. Preliminary Statement

A marked increase in the use of resistance welding machines has occurred during the past several years. New applications of resistance welding are being developed, and more of them are expected in the future. This increased use of resistance welders, with the resultant light flicker that may occur, has added to the voltage regulation problems of the utility engineer.

The usual resistance welder puts a large kva load on the power system serving the welder. The welder load is applied at frequent intervals and for periods ranging from one or two cycles to several seconds in length. This on-and-off sequence of welder operation causes a corresponding fluctuation in voltage on the power system; if these changes in voltage are large enough, other customers on the system adjacent to the welder will be subjected to objectionable light flicker.

Light flicker is much more noticeable or objectionable at some frequencies of fluctuation than at others. If the load is such that the flicker occurs at random intervals, but at such a rate that there are several fluctuations per minute, the voltage change must be limited to 1.0-1.5 volts on a 120-volt base if objectionable lighting flicker is to be avoided. The requirements to avoid objectionable flicker with a cyclic voltage variation are even more severe. If the cyclic frequency at which the welder load is switched off and on is about six times per second, the maximum allowable voltage drop is of the order of 0.5 volt on a 120-volt base. The above frequency of voltage fluctuation is typical of that produced by many seam welders which join two pieces of metal by making a series of evenly spaced welds in the form of a seam.

The power company engineer must be able to determine accurately the variation in voltage caused by welder loads in order to provide service at the lowest cost. Generally these problems have been handled satisfactorily with considerable ingenuity in devising methods to keep the cost of service reasonable. However, in some cases involving single-phase welders, the voltage fluctuations after the installation of a welder were appreciably lower than had been calculated. It was reasoned by certain power company engineers that this discrepancy was caused by large motor loads adjacent to the welder. A study of this problem and methods of calculation have previously been published in Bulletin No. 392 of the University of Illinois Engineering Experiment Station. As an extension of the original problem concerning only single-phase welders, the effect of adjacent motor loads on the voltage fluctuations caused by three-phase welders has been studied. The results of this study as reported herein show somewhat the same end result as for the single-phase problem; that is, the adjacent motor loads reduce the voltage fluctuations caused by three-phase resistance welders. This is particularly true for those welders having very short "off" periods. However, the principal reason for the improvement due to the motors is not a result of the same effects which were instrumental in the improvement of the voltage fluctuation caused by single-phase welders. This will become apparent when this report is compared with Bulletin No. 392.

2. Acknowledgments

This investigation was supported by funds contributed by the Utilities Research Commission, Chicago, Illinois. Mr. M. S. Oldacre, Director of Research for the Commission, appointed the following Advisory Committee to cooperate in the study:

R. O. Askey, Chairman, Public Service Company of Northern Illinois
F. G. Mueller, Commonwealth Edison Company
H. E. Smith, Commonwealth Edison Company
R. E. Young, Public Service Company of Northern Illinois

The Advisory Committee was very active and made many valuable contributions during the progress of the work.

The investigation was carried on as a part of the work of the Engineering Experiment Station of the University of Illinois under the general administrative direction of Dean W. L. Everitt, Director of the Station.

Acknowledgment is made to Mr. V. K. Kraybill of the Public Service Company of Northern Illinois for the development of a device, mentioned in the footnote of page 25, which will measure the effective value of a rapidly changing voltage. Without this device measurement of experimental values of voltage fluctuation would have been extremely difficult.

Acknowledgment is also made to Mr. J. L. Solomon and Mr. F. W. Jaksha of the Sciaky Brothers Company of Chicago, Illinois, for their suggestions and advice during the course of the investigation.

II. ANALYTICAL DEVELOPMENT OF VOLTAGE CHANGE EQUATIONS

3. The Nature of the Three-Phase Welder Load

The oscillogram of Fig. 1 shows typical waveforms for the three line currents required for the operation of a three-phase welder. It will be noted that the peak values of a given line current increase in value from one cycle to the next and that the waveforms of the three line currents are not exactly the same. Futhermore, the line currents are zero periodically since the welder load is alternately removed from the system for periods from a fraction of a cycle to several cycles and then reconnected for a time interval determined by the particular welding job.* It is therefore apparent that the calculation of the voltage changes produced by a three-phase welder load cannot be made by direct application of the usual methods applied to balanced three-phase sinusoidal loads.

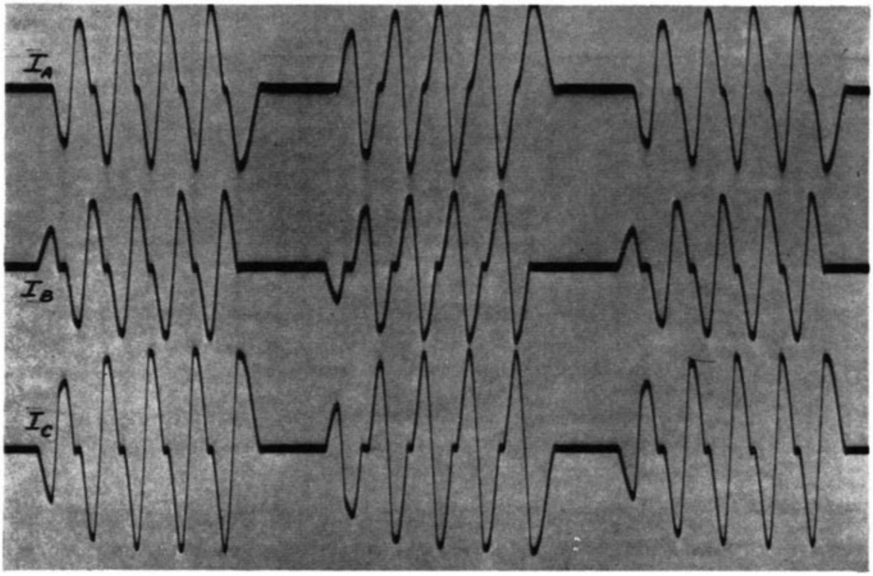


Fig. 1. Oscillogram of Typical Three-Phase Welder Line Currents

* For a detailed discussion of three-phase welder operation and waveforms of various currents and voltages see "Circuit Analysis of Frequency-Changer Welders," by W. K. Boice, Welding Journal, Volume 28, 1949, pages 946-956.

4. Assumptions Made in Development of Equations

In order to provide an analytical solution to the problem under discussion certain assumptions are made concerning the three-phase welder load. These assumptions permit the effect of the welder load to be evaluated in terms of a more easily handled sinusoidal load. The reasonableness of the assumptions discussed below is further verified in a later section on experimental data.

It is first assumed that the "on" period of the welder is relatively long and that the change in peak line currents near the end of the conduction period is small. If these conditions are sufficiently satisfied, the system will be operating at the end of the "on" period under the same conditions of current and voltage as would be caused by a steady-state load of the same current and wave shape as the last cycle of welder current. For these conditions to exist, the welder "on" period would probably have to be at least three cycles of the line frequency, a situation usually found in practice. When the above conditions are satisfied, the changes in voltage when the welder "off" period occurs will be the same as those that would occur if the above-mentioned steady-state load were removed from the system. Thus the problem of calculating the voltage changes has been reduced to one of finding the changes which occur when a steady-state load is removed from the system.

This steady-state load is a non-sinusoidal one. However, the voltage changes produced by a non-sinusoidal load, which is not too distorted, can be found within a close approximation by considering only the fundamental component of the non-sinusoidal current. When a non-sinusoidal load is connected at a given point in the system, the voltage changes from a sinusoidal condition to a non-sinusoidal one. The fundamental component of voltage is calculated using the fundamental component of current. Each of the harmonic voltages existing after the load is connected are calculated using the corresponding harmonic component of load current. Inasmuch as these harmonic currents are assumed to be only a fraction of the fundamental current, the harmonic voltages are small compared to the fundamental voltage. Since the rms voltage with the load connected is found from the square root of the sum of the squares of the fundamental and harmonic voltages, the rms voltage will be determined almost entirely by the fundamental voltage. Thus the change in rms voltage can be calculated to a very close approximation using only the change in the fundamental voltage. It follows therefore that the effect on the system voltages of a non-sinusoidal load of low harmonic content can be predicted very closely by considering the effect of a sinusoidal load current of a value equal to the fundamental component of the non-sinusoidal load.

The effect on the system of removing the load is handled mathematically by considering that there is added to the system a load which requires a current that is the negative of the current to the welder load at the end of the "on" period. The sum of these two currents is obviously zero; this method thus provides the same effect as disconnecting the load.

Summarizing, the method used is to consider the welder load as a balanced sinusoidal three-phase load. The impedance of this load is taken as such a value that the current to the sinusoidal load will be the same as the fundamental component of the actual welder current during the last cycle of the "on" period. A load which requires exactly the negative current of the sinusodial load is considered to be added to the system at the end of the "on" period so as to reduce the total load to zero. The effect on the system voltages of adding this negative load is computed by the method of superposition.

5. Equations for Voltage Change at the Welder

Let it be assumed that Fig. 2 represents one phase of the balanced three-phase system to which the welder is connected and that the

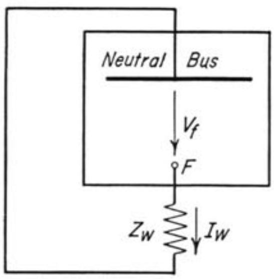


Fig. 2. Circuit Diagram for One Phase of a Balanced Three-Phase System which is Supplying a Balanced Three-Phase Load Connected at Point F

equivalent sinusoidal load is connected at F long enough for steady-state conditions to be reached. Let $\overline{V}_f$ be the neutral-to-line voltage rise at F with the load connected and $\overline{V}_F$ be the neutral-to-line voltage rise at F when the load is removed. Let $\bar{I}_W$ be the line current to the load.

If the load is removed from the system by considering that a current $-\bar{I}_W$ is required to flow from the system, then by superposition

$$\overline{V}_F = \overline{V}_f - (-\bar{I}_W)\,\overline{Z}_f \tag{1}$$

where $\overline{Z}_f$ is the impedance per phase from F to the neutral bus with all generated voltages made equal to zero.

Also

$$\bar{I}_W = \frac{\bar{V}_f}{\bar{Z}_W} \tag{2}$$

where $\bar{Z}_W$ is the per phase impedance of the welder equivalent load. Then

$$\bar{V}_F = \bar{V}_f + \frac{\bar{V}_f \bar{Z}_f}{\bar{Z}_W} = \bar{V}_f\left(1 + \frac{\bar{Z}_f}{\bar{Z}_W}\right) \tag{3}$$

or

$$\frac{V_F}{V_f} = \left| 1 + \frac{\bar{Z}_f}{\bar{Z}_W} \right| \tag{4}$$

In Eq. 4 the vertical bars indicate the magnitude of the complex quantity. This is done since usually only the magnitude of the voltage ratio is of importance. If the change in voltage is desired in percent, then

$$\text{Percent change in voltage} = 100\left(\frac{V_F}{V_f} - 1\right) \tag{5}$$

If V_F/V_f has a value of 1.015, a change in voltage of 1.5 percent will occur when the load is removed.

6. Change in Voltage at the Primary of the Welder Supply Transformer Bank

Generally a welder is served by a supply system which is similar to that in Fig. 3. The point P is the location at which the customer's step-down transformer bank is connected to the utility lines. The motor load at P represents the industrial motor load in the same area served from the same feeder. The motor load at F is the motor load in the customer's plant on the same bus as the welder. The motor load at either P or F, of course, may not exist.

The change in voltage at the welder, though of importance in many problems, is not usually as important to the utility engineer as the change in voltage at the point P. Any change in voltage at this point affects all the other customers connected to the same supply circuit.

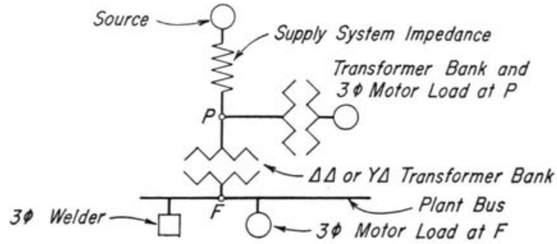


Fig. 3. Circuit I. A Typical System for Supplying a Three-Phase Welder

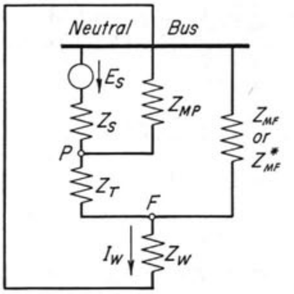


Fig. 4. Equivalent Circuit for the System Shown in Fig. 3 during the Time that the Welder is Connected to the System

Circuit I: Motors at P and F. Figure 4 is the single-line equivalent circuit for the system of Fig. 3. The following definitions apply to the equivalent circuit:

$\bar{E}_S$ = excitation voltage rise of source, neutral-to-line

$\bar{Z}_S$ = impedance per phase of supply system to the point P

$\bar{Z}_T$ = impedance per phase of transformer serving the welder load (equivalent $Y-Y$)

$\bar{Z}_W$ = impedance per phase of the equivalent welder load (equivalent Y value)

$\bar{Z}_{MP}$ = impedance per phase of motor load and transformer connected at P (equivalent $Y-Y$)

$\bar{Z}_{MF}$ = impedance per phase of the motor load at F (equivalent Y value)

$\bar{Z}_{MF}{}^*$ = impedance per phase of the motor load at F under steady-state conditions (equivalent Y value)

Assume that the equivalent circuit of Fig. 4 is operating under steady-state conditions with the equivalent welder load connected to the system. Let $\bar{V}_p$ and $\bar{V}_f$ be the neutral-to-line voltage rises at P and F with the equivalent sinusoidal load connected. Let $\bar{V}_P$ and $\bar{V}_F$ be the neutral-to-line voltage rises at P and F after the load is removed.

With the load connected

$$\bar{V}_p = \bar{V}_f + \frac{\bar{V}_f}{\bar{Z}_{MF}{}^*}\bar{Z}_T + \frac{\bar{V}_f}{\bar{Z}_W}\bar{Z}_T \tag{6}$$

$$\bar{V}_p = \bar{V}_f\left(1 + \frac{\bar{Z}_T}{\bar{Z}_{MF}{}^*} + \frac{\bar{Z}_T}{\bar{Z}_W}\right) \tag{7}$$

The load is removed from the system by requiring that an additional current $-\bar{I}_W$ flow from the system. The component of this current in

$\overline{Z}_S$ is found, and using this component $\overline{V}_P$ may be calculated. The machine impedances to be used should be those presented to the change in current. Thus

$$\Delta\overline{I}_S = (-\overline{I}_W)\frac{\overline{Z}_{MF}}{\overline{Z}_{MF} + \overline{Z}_T + \dfrac{\overline{Z}_{MP}\overline{Z}_S}{\overline{Z}_{MP} + \overline{Z}_S}} \cdot \frac{\overline{Z}_{MP}}{\overline{Z}_{MP} + \overline{Z}_S} \tag{8}$$

where

$$\overline{I}_W = \frac{\overline{V}_f}{\overline{Z}_W} \tag{9}$$

By the principle of superposition

$$\overline{V}_P = \overline{V}_p - \Delta\overline{I}_S\overline{Z}_S \tag{10}$$

Substituting Eqs. 7, 8, and 9 in Eq. 10

$$\overline{V}_P = \overline{V}_f\left(1 + \frac{\overline{Z}_T}{\overline{Z}_{MF}^*} + \frac{\overline{Z}_T}{\overline{Z}_W}\right) + \frac{\overline{V}_f}{\overline{Z}_W} \cdot \frac{\overline{Z}_{MF}\overline{Z}_{MP}\overline{Z}_S}{(\overline{Z}_S + \overline{Z}_{MP})(\overline{Z}_{MF} + \overline{Z}_T) + \overline{Z}_{MP}\overline{Z}_S} \tag{11}$$

Using Eqs. 11 and 7 there is obtained

$$\frac{\overline{V}_P}{\overline{V}_p} = \frac{\left(1 + \dfrac{\overline{Z}_T}{\overline{Z}_{MF}^*} + \dfrac{\overline{Z}_T}{\overline{Z}_W}\right) + \dfrac{\overline{Z}_{MF}\overline{Z}_{MP}\overline{Z}_S}{\overline{Z}_W[(\overline{Z}_S + \overline{Z}_{MP})(\overline{Z}_{MF} + \overline{Z}_T) + \overline{Z}_{MP}\overline{Z}_S]}}{1 + \dfrac{\overline{Z}_T}{\overline{Z}_{MF}^*} + \dfrac{\overline{Z}_T}{\overline{Z}_W}} \tag{12}$$

This simplifies to give

$$\frac{\overline{V}_P}{\overline{V}_p} = 1 + \frac{\overline{Z}_{MF}^*\overline{Z}_{MF}\overline{Z}_{MP}\overline{Z}_S}{[\overline{Z}_{MF}^*\overline{Z}_W + \overline{Z}_T\overline{Z}_W + \overline{Z}_{MF}^*\overline{Z}_T][(\overline{Z}_S + \overline{Z}_{MP})(\overline{Z}_{MF} + \overline{Z}_T) + \overline{Z}_{MP}\overline{Z}_S]} \tag{13}$$

Usually only the magnitude of the change in voltage is desired and the results may be expressed as

$$\frac{V_P}{V_p} = \left| 1 + \frac{\overline{Z}_{MF}^*\overline{Z}_{MF}\overline{Z}_{MP}\overline{Z}_S}{[\overline{Z}_{MF}^*\overline{Z}_W + \overline{Z}_T\overline{Z}_W + \overline{Z}_{MF}^*\overline{Z}_T][(\overline{Z}_S + \overline{Z}_{MP})(\overline{Z}_{MF} + \overline{Z}_T) + \overline{Z}_{MP}\overline{Z}_S]} \right| \tag{14}$$

where the vertical bars indicate the magnitude of the expression enclosed.

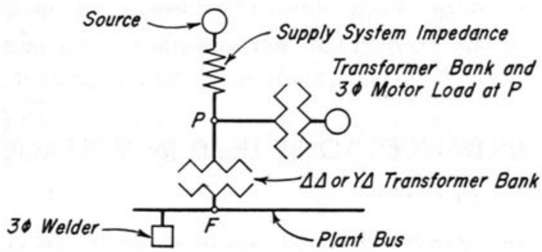


Fig. 5. Circuit II. A Second Typical System for Supplying a Three-Phase Welder

Circuit II: Motors at P only. In this case (Fig. 5) the voltage ratio V_P/V_p may be found from Circuit I by letting $\bar{Z}_{MF}$ and $\bar{Z}_{MF}^*$ be infinite since there are no motors at F. Using these values in Eq. 14 and simplifying, the following is obtained

$$\frac{V_P}{V_p} = \left| 1 + \frac{\bar{Z}_{MP}\bar{Z}_S}{(\bar{Z}_W + \bar{Z}_T)(\bar{Z}_S + \bar{Z}_{MP})} \right| \tag{15}$$

Circuit III: Motors at F only. The voltage ratio V_P/V_p can be found for the case of Fig. 6 by again simplifying Eq. 14. Substituting $\bar{Z}_{MP} = \infty$ the following result is obtained

$$\frac{V_P}{V_p} = \left| 1 + \frac{\bar{Z}_{MF}^*\bar{Z}_{MF}\bar{Z}_S}{(\bar{Z}_{MF}^*\bar{Z}_W + \bar{Z}_T\bar{Z}_W + \bar{Z}_{MF}^*\bar{Z}_T)(\bar{Z}_{MF} + \bar{Z}_T + \bar{Z}_S)} \right| \tag{16}$$

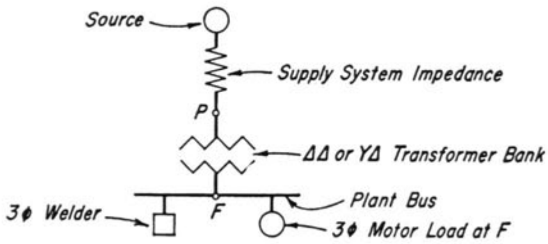


Fig. 6. Circuit III. A Third Typical System for Supplying a Three-Phase Welder

III. MOTOR IMPEDANCES TO BE USED IN VOLTAGE CHANGE EQUATIONS

The impedance $\bar{Z}_{MF}^*$ which appears in some of the voltage change equations is always that impedance of the motor load at F as determined from the voltage, current and power to the motor load at the end of the welder "on" period. For an induction motor this will be just the usual steady-state impedance corresponding to the terminal voltage and the particular load on the motor. For a synchronous motor the impedance determined as above is rather fictitious since the synchronous motor is usually represented by an excitation voltage in series with the synchronous impedance. The impedance $\bar{Z}_{MF}^*$ for the synchronous machine will depend on the value of the field current as well as the terminal voltage and the load on the motor.

The other impedances to be used will depend to a large extent on the "off" period of the three-phase welder load. The welder loads are therefore classified in terms of the length of the "off" period. The first group, Type 1 loads, includes those welder loads for which the "off" period is long enough for the system to reach steady-state conditions during the "off" period. Type 2 loads are those for which the "off" period is so short that steady-state conditions are not reached during the "off" period. In general, the procedure in calculating the voltage change is to use an impedance which corresponds to the impedance presented to the change in current at the time the end of the "off" period is reached.

7. Induction Motors

For Type 1 loads the usual steady-state impedance of the motor, as determined from the equivalent circuit for the particular load, should be used. Since steady-state conditions exist at the end of the "off" period, the impedance presented to the component of the current $-\bar{I}_W$ in the motor is just the steady-state impedance of the motor.

For Type 2 loads an impedance corresponding to the conditions at the end of the "off" period should be used for the motor. For "off" periods which are extremely short, nearly zero time, an impedance equal to the blocked-rotor impedance should be used for the motor. It is well known that the initial impedance presented to a change in current by an induction motor is equal to the blocked-rotor impedance of the motor. (This impedance might be compared to the sub-transient impedance of a synchronous machine.) Values of the steady-state and blocked-rotor impedances for induction motors in the range of 10-500 hp are listed in Table 8, page 37.

The "off" period cannot actually be of zero duration, and thus the voltage change for a particular case would be intermediate between the two values calculated using steady-state and blocked-rotor impedances. Approximate methods of finding the voltage change in such a case are given in Chapter V.

8. Synchronous Motors

As in the case of the induction motor, the steady-state impedance of a synchronous motor is used for Type 1 welder loads. This would be the usual synchronous reactance for a cylindrical-rotor machine. In the case of a salient-pole machine the direct-axis reactance of the motor should be used as the synchronous reactance of an equivalent cylindrical-rotor machine. The slight additional accuracy of considering the saliency of the machine would not be warranted for a problem of this type.

For Type 2 loads the impedance that should be used would depend on the length of the welder "off" period. For an extremely short "off" period the sub-transient reactance of the motor should be used. For somewhat longer "off" periods the transient reactance should be used.

A detailed analysis of the voltage change for any "off" period, such as that of Chapter V concerning the induction motor, could be made for the synchronous motor. However, the great majority of problems involve motor loads which are predominately induction motors and, therefore, it did not seem worth while to make such an analysis for the very few cases involving large synchronous motor loads.

Average values of small synchronous motor constants are given in Table 9, page 37. The impedances given in this table permit the calculation of the voltage change for long or very short "off" periods. The range of motor time constants given can be used to estimate the voltage change for other "off" periods.

IV. EXPERIMENTAL DATA

The experimental portion of this project was done in two separate parts. The first group of tests was made with a balanced three-phase sinusoidal load on the system in order to check the analytical development which is given in Chapter II. Another reason for first using this type of load was that it was relatively easy to provide the controls to switch such a load on and off. After the first series of tests had established the help of the motor load and verified the method of calculation for the sinusoidal load, the next step was a test with a load approximating the three-phase welder load. An example of the line current of this simulated three-phase welder has already been shown in Fig. 1. These two series of tests provide information regarding the reasonableness of the assumptions that were made in solving the problem analytically. They also provide information regarding the motor impedances that should be used with various "off" times.

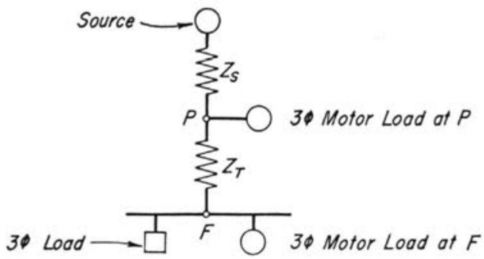


Fig. 7. Circuit for Experimental Tests Involving a Balanced Sinusoidal Three-Phase Load

9. Balanced Sinusoidal Three-Phase Load

The first series of tests was conducted using a balanced three-phase resistance load. This was switched off and on for certain periods of time by two thyratron tubes. It was possible to adjust these "on" and "off" periods to simulate the "on" and "off" periods that might be encountered in various welding operations.

The circuit diagram of the whole system is shown in Fig. 7. With the motor load at F voltage changes were measured at both P and F; with the motor load at P voltage changes were measured at P only.

Method of Measuring Voltage Changes. The measurement of the voltage changes was made by measurement of the peak values of voltage from oscillograms. In Fig. 8 is shown the envelope of peak values that

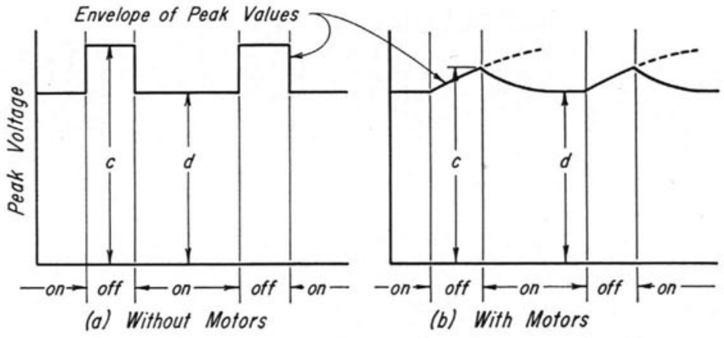


Fig. 8. Envelope of Peak Voltage Values that Occur when the Three-Phase Load of Fig. 7 is Switched Off and On

occur when the load is switched off and on. Part *a* shows this envelope when no motors are connected to the system and part *b* shows the effect on the envelope when a motor load is connected.

The ratio of the highest voltage peak during the "off" period to the voltage at the end of the "on" period was determined for each of the tests. From the figure this can be seen to be c/d. It can also be seen from part *b* of the figure that as the "off" time increases the ratio c/d becomes larger until a steady-state value is obtained. This shows that the change in voltage is smaller the shorter is the "off" period.

Typical Oscillograms and Calculations. The typical oscillograms which are shown in Fig. 9 apply to the circuit of Fig. 7 with no motors connected at point F. All of the voltages shown on the oscillograms are for the point P and the currents shown are the load line currents. The test conditions for each of the oscillograms are indicated in Table 1. Table 2 shows values of the voltage ratio V_P/V_p that were determined by

Table 1
Test Conditions for the Oscillograms Shown in Fig. 9*

Oscillogram #	*Motor Load* *at P*	*Cycles* On	Off	*Remarks*
4-1	Off	5½	½	V_{AB} at P
13-1	On	5½	½	V_{AB} at P
53-1	Off	5½	½	Load Currents I_A, I_B, and I_C
34-1	Off	5	4	V_{AB} at P
43-1	On	5	4	V_{AB} at P
55-1	Off	5	4	Load Currents I_A, I_B, and I_C

* There was no motor connected at F in this series of tests. When the motor was connected at P, it was operated at no load.

Table 2
Measured and Calculated Voltage Ratios for the Oscillograms Shown in Fig. 9

Oscillogram #	"c" *Amplitude at End of Off Period*	"d" *Amplitude at End of On Period*	c/d or V_P/V_p	*Calculated* V_P/V_p
4-1	31.39	29.06	1.079	1.071
13-1	29.29	27.99	1.047	1.045
34-1	30.88	28.68	1.077	1.071
43-1	29.54	27.70	1.066	1.066

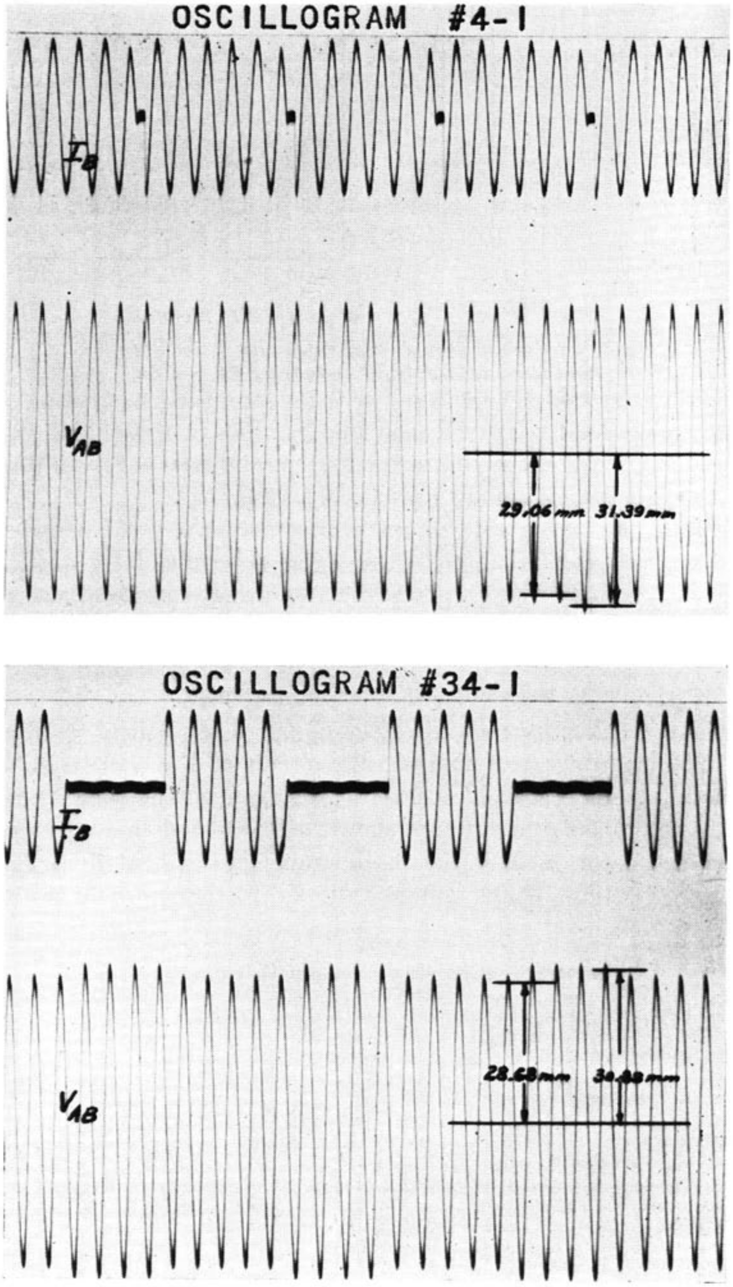
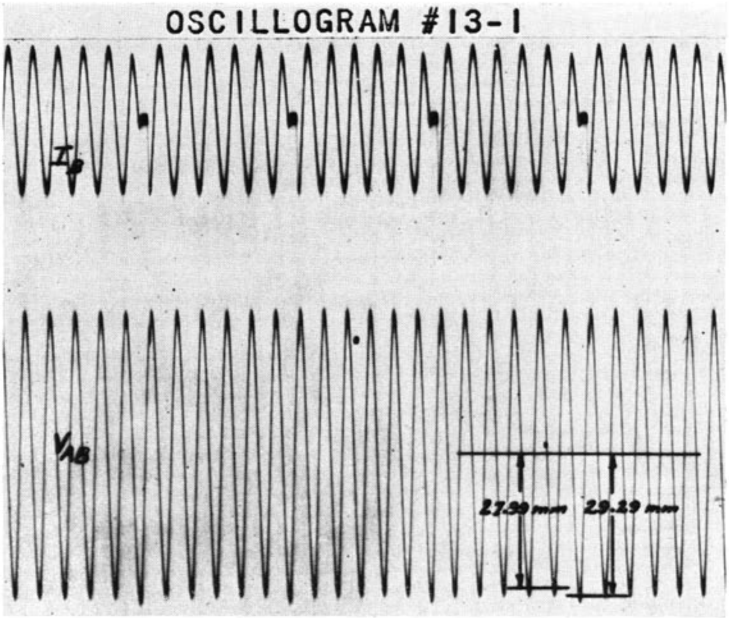


Fig. 9. Oscillograms for the Test Conditions Specified in Table 1



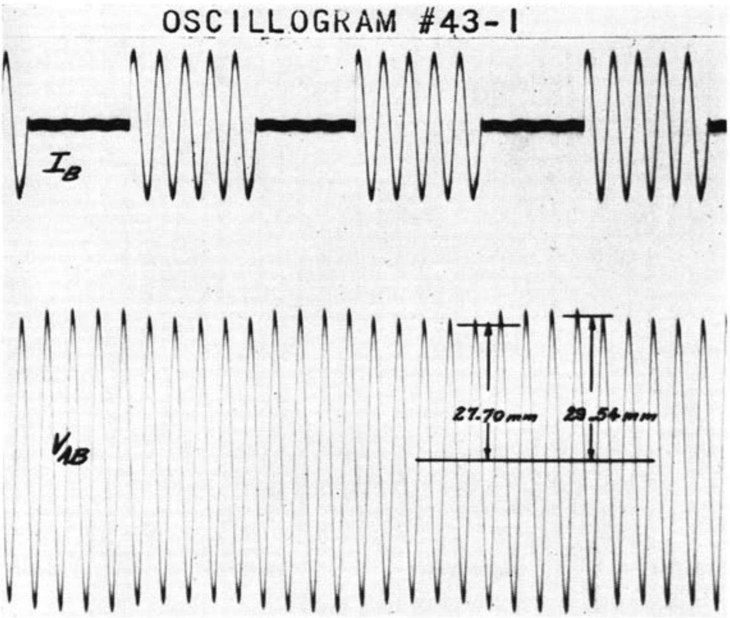
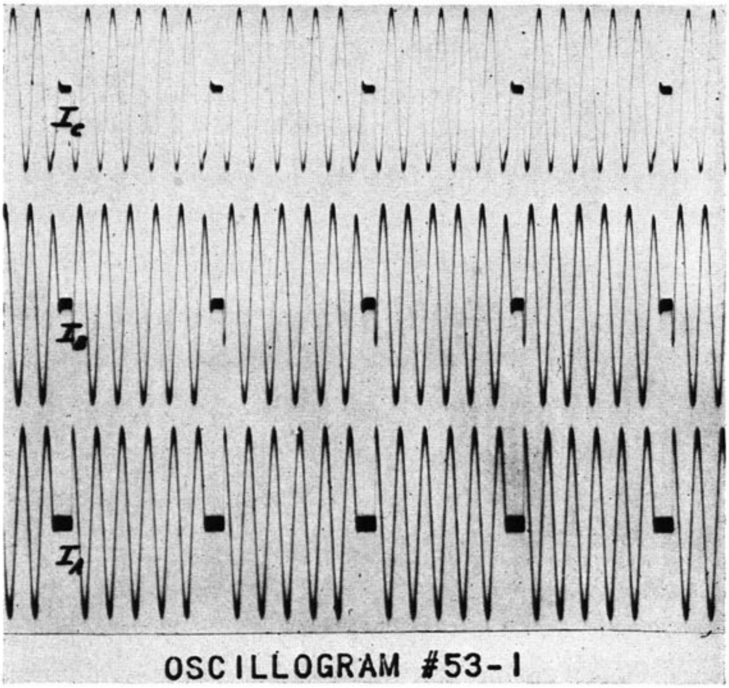


Fig. 9. Oscillograms for the Test Conditions Specified in Table 1 (continued)



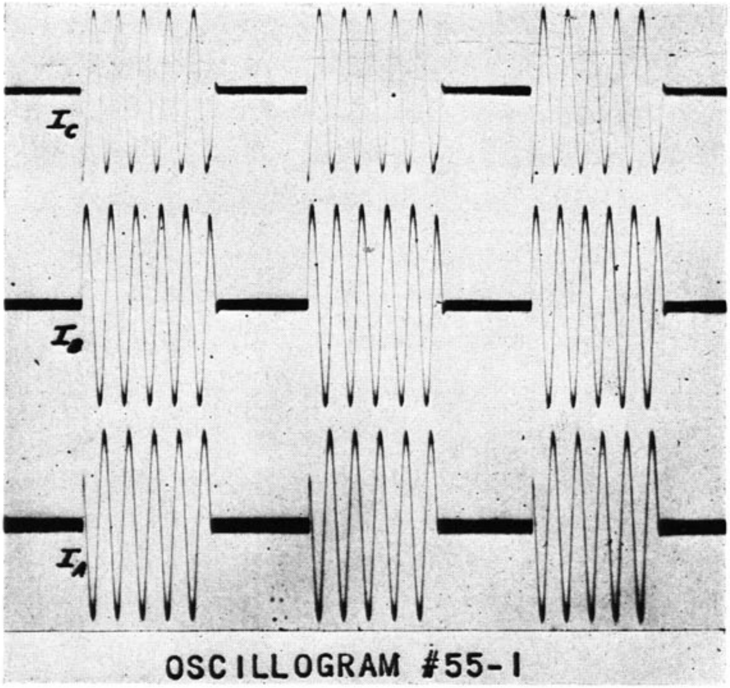


Fig. 9. Oscillograms for the Test Conditions Specified in Table I (concluded)

measurements made on the oscillograms of Fig. 9 and values of V_P/V_p calculated by the method described in Chapters II and III.

SAMPLE CALCULATIONS FOR TABLE 2:

Data for System of Fig. 7:	Induction Motor Data:
$\bar{Z}_S = 0.74+j\,.515$ ohms	Rating: 7½ hp, 220 v, 1135 rpm, 3ϕ
$\bar{Z}_T = 0.64+j\,.475$ ohms	No-Load Impedance $= 2.14 + j\,13.60$ ohms
$\bar{Z}_W = 10.18+j\;0$ ohms	Blocked-Rotor Impedance $= 1.15 + j\,1.54$ ohms

Note: The impedance $\bar{Z}_W$ was kept fixed at the value given for all tests.

For oscillograms 4-1 and 34-1 (no motors connected):
With no motor at P in Fig. 5, Eq. 15 becomes

$$\frac{V_P}{V_p} = \left| 1 + \frac{\bar{Z}_S}{\bar{Z}_T + \bar{Z}_W} \right| = \left| 1 + \frac{0.74 + j\,0.515}{0.64 + j\,0.475 + 10.18} \right|$$

$$= \left| 1 + \frac{0.904 \angle 34.9^\circ}{10.83 \angle 2.5^\circ} \right|$$

$$\frac{V_P}{V_p} = \left| 1 + .0835 \angle 32.4^\circ \right| = 1.071$$

For oscillogram 13-1 (motor at P):
Using Eq. 15 with the blocked-rotor impedance for $\bar{Z}_{MP}$ gives

$$\frac{V_P}{V_p} = \left| 1 + \frac{\bar{Z}_{MP}\bar{Z}_S}{(\bar{Z}_T + \bar{Z}_W)\,(\bar{Z}_{MP} + \bar{Z}_S)} \right|$$

$$= \left| 1 + \frac{(0.74 + j\,0.515)\,(1.15 + j\,1.54)}{(0.64 + j\,0.475 + 10.18)\,(1.15 + j\,1.54 + 0.74 + j\,0.515)} \right|$$

$$\frac{V_P}{V_p} = \left| 1 + .0575 \angle 38.3^\circ \right| = 1.045$$

For oscillogram 43-1 (motor at P):
Using Eq. 15 with the no-load or steady-state impedance for $\bar{Z}_{MP}$ yields

$$\frac{V_P}{V_p} = \left| 1 + \frac{\bar{Z}_{MP}\bar{Z}_S}{(\bar{Z}_T + \bar{Z}_W)\,(\bar{Z}_{MP} + \bar{Z}_S)} \right|$$

$$= \left| 1 + \frac{(2.14 + j\,13.6)\,(0.74 + j\,0.515)}{(0.64 + j\,0.475 + 10.18)\,(2.14 + j\,13.6 + 0.74 + j\,0.515)} \right|$$

$$\frac{V_P}{V_p} = \left| 1 + 0.80 \angle 34.9^\circ \right| = 1.066$$

Comparison of Measured and Calculated Voltage Ratios for the Complete Series of Tests Using the Balanced Sinusoidal Three-Phase Load. Table 3 shows a comparison of measured and calculated values for the ratios V_P/V_p and V_F/V_f for three different "off" and "on" conditions. This comparison shows a good agreement for all tests in which no motor load was connected. For the series of tests involving a ½-cycle "off" time and a 5½-cycle "on" time the measured values show a good agreement with the values calculated using the blocked-rotor impedance for the motor load. For the series of tests involving a 4-cycle "off" time and a 5-cycle "on" time, the values calculated with steady-state motor impedances show a good agreement with measured values. The observed values for the tests with a 1-cycle "off" time and a 6-cycle "on" time are numerically between those calculated using blocked rotor and steady-state impedances. This is the expected condition since the "off" period was not quite long enough for steady-state conditions to be attained. In order to predict the actual value of the voltage ratio in such a situation the "off" time in relation to the motor time constant would have to be known. This is discussed more fully in Chapter V.

Table 3
Measured and Calculated Voltage Ratios for All of the Experimental Tests Using a Balanced Three-Phase Load

Motor Location	*Voltage Measured or Calculated at*	*Phase*	*Measured Values*				*Calculated Values*		
			Without Motors	With Motors			Without Motors	With Motors	
				½~ Off Period	1~ Off Period	4~ Off Period		Using Blocked-Rotor Impedance	Using Steady-State Impedance
		AB	1.156	1.070	1.102	1.126			
F	*F*	BC	1.150	1.064	1.100	1.130	1.141	1.068	1.113
		CA	1.140	1.059	1.079	1.113			
		AB	1.079	1.034	1.044	1.062			
F	*P*	BC	1.065	1.037	1.036	1.053	1.071	1.032	1.060
		CA	1.067	1.024	1.044	1.056			
		AB	1.079	1.047	1.061	1.066			
P	*P*	BC	1.065	1.050	1.054	1.068	1.071	1.045	1.066
		CA	1.067	1.043	1.049	1.067			

10. Simulated Three-Phase Welder Load

The second series of tests was conducted using as a load a simulated three-phase welder. The line currents to this load were similar to those required by an actual three-phase welder. An oscillogram of these line currents has already been given in Fig. 1. The tests were conducted using the system shown in Fig. 10. All of the voltage changes were measured at point *F*. The series included tests for several "off" and "on" conditions both with and without the motor load connected.

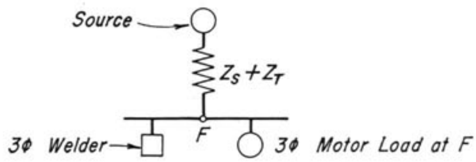


Fig. 10. Circuit for Experimental Tests Involving a Simulated Three-Phase Welder Load

Method of Measuring Voltage Changes. Since the load currents were not sinusoidal, the voltages at F could no longer be expected to be sinusoidal. Thus the measurement of peak voltages to obtain rms voltages could no longer be done. The measurements were made by an instrument which was developed for just such a problem — the measurement of a rapidly changing non-sinusoidal voltage. The instrument has been previously described at an AIEE Welding Conference.*

The indications of the instrument are in the form of a series of pulses on an oscillogram. Each pulse corresponds to the rms value of one half-cycle of the voltage in question and may be determined from calibration of the instrument and oscillograph combined. Figure 11 shows the record

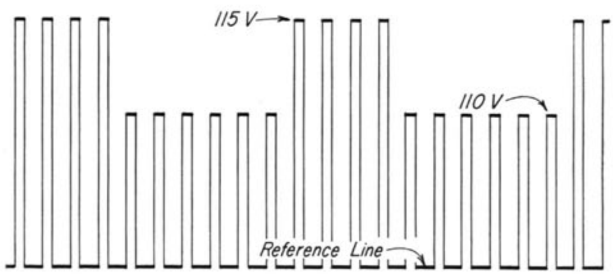


Fig. 11. Typical Record of Instrument Used to Measure Rapidly Changing Non-Sinusoidal Voltages

that would be obtained from a voltage that is changing abruptly from one value to another, say 110 to 115 volts. The instrument has a suppressed zero, so that the useful range is a few percent of the nominal value of the voltage being measured.

In much the same way as was done for the other series of tests, the highest voltage recorded during the "off" period and the voltage at the end of the "on" period were used to form the voltage ratio.

Typical Oscillograms and Calculations. Four typical oscillograms from this series of tests are shown in Fig. 12. The test conditions for each of the oscillograms are given in Table 4.

* New Devices for Measurement of Welder Voltage Drops on a Power System, by Marvin Fisher, Jr., V. H. Kraybill, R. E. Young. Presented at AIEE Welding Conference, April, 1952, Detroit, Michigan.

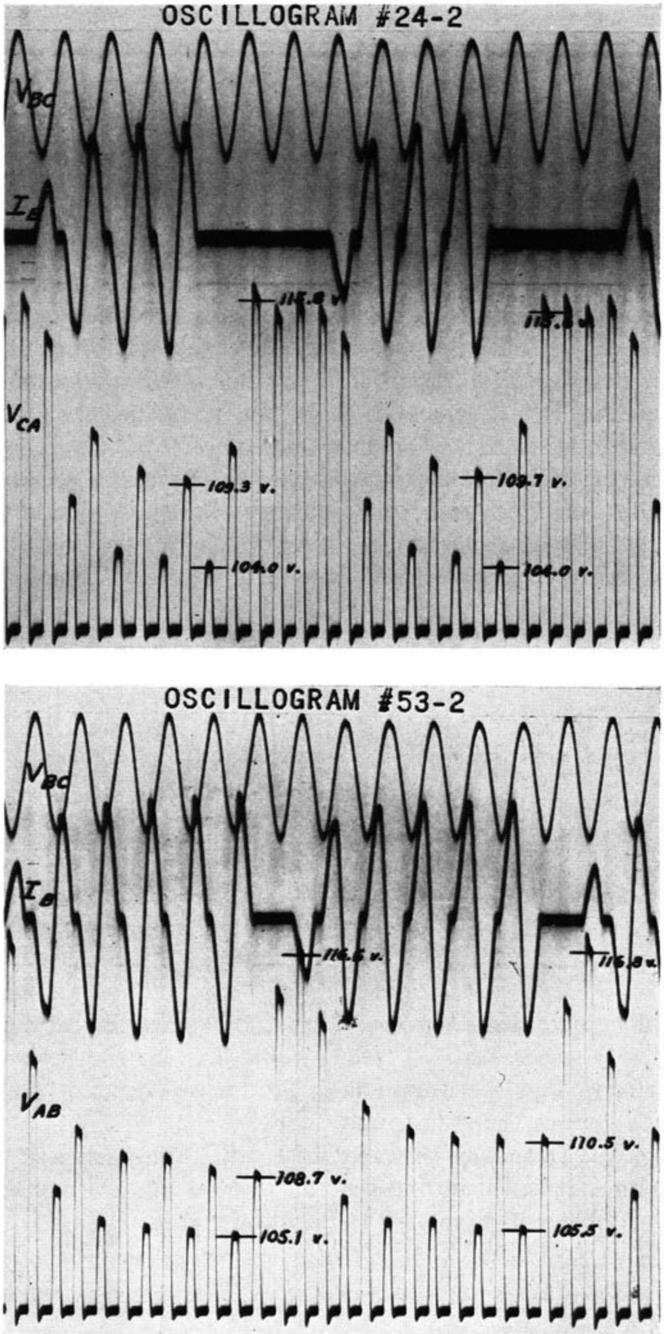
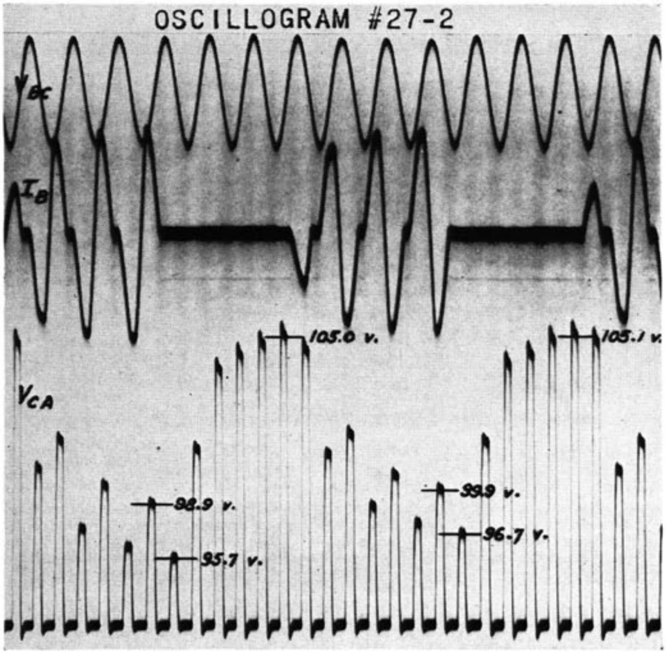


Fig. 12. Oscillograms for the Test Conditions Specified in Table 4



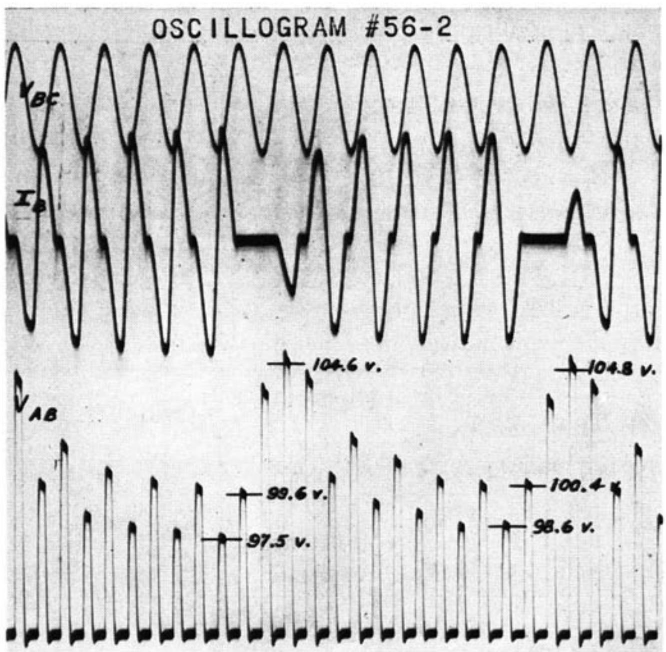


Fig. 12. Oscillograms for the Test Conditions Specified in Table 4 (concluded)

Table 4
Test Conditions for the Oscillograms Shown in Fig. 12*

Oscillogram #	*Motor Load at F*	*On Time Approx. Cycles*	*Off Time Approx. Cycles*	*Voltage Measured*
24-2	Off	3½	3	V_{CA}
27-2	On	3½	3	V_{CA}
53-2	Off	5½	1	V_{AB}
56-2	On	5½	1	V_{AB}

* When the motor was connected at *F*, it was operated at no load.

Table 5
Measured and Calculated Voltage Ratios for the Oscillograms Shown in Fig. 12

Oscillogram #	*Voltage at End of On Period**	*Voltage at End of Off Period*	*Measured Ratio*	*Calculated Ratios*	
				Using Blocked-Rotor Impedance	Using Steady-State Impedance
24-2	106.85	115.6	1.083	No Motor Connected	1.082
27-2	98.3	105.1	1.070	1.046	1.072
53-2	108.0	116.8	1.082	No Motor Connected	1.079
56-2	99.5	104.8	1.052	1.046	1.078

* The effective value of a voltage whose positive and negative half-cycles have different effective values can be shown to be very nearly equal to the average of the positive and negative half-cycle effective values provided the difference is not too large. This is done for the last two half-cycles of voltage during the "on" period. The voltages for the "on"-"off" sequence near the right side of each oscillogram are tabulated.

The pulse on the oscillogram corresponding to each half-cycle of voltage has been marked with its corresponding value taken from the calibration curve for the test. Table 5 gives a comparison of the values taken from the oscillograms and values calculated according to the equations of Chapter II.

Sample Calculations for Table 5:

In Fig. 10

$$\bar{Z}_S + \bar{Z}_T = 0.443 + j\,1.13 = 1.21\,\underline{/69.1^\circ}\text{ ohms}$$

$$\bar{Z}_{MF} = 4.05 + j\,12.85\text{ ohms (Steady-State)}$$

$$\bar{Z}_{MF} = 0.85 + j\,1.42\text{ ohms (Blocked-Rotor)}$$

Equation 4 is used to calculate the voltage ratios in each of the following sample calculations.

For oscillogram 24-2:

$\bar{Z}_W = 13.34\,\underline{/42.8^\circ}$ ohms

With no motor load $\bar{Z}_f = \bar{Z}_T + \bar{Z}_S$. Thus

$$\frac{V_F}{V_f} = \left| 1 + \frac{\bar{Z}_f}{\bar{Z}_W} \right| = \left| 1 + \frac{1.21\,\underline{/69.1^\circ}}{13.34\,\underline{/42.8^\circ}} \right| = 1.082$$

For oscillogram 27-2:

$\bar{Z}_W = 14.2\,\underline{/42.8^\circ}$ ohms

$\bar{Z}_f$ is determined from the parallel combination of $\bar{Z}_S + \bar{Z}_T$ and $\bar{Z}_{MF}$.

Using blocked-rotor impedance

$$\bar{Z}_f = \frac{(1.21 \underline{/69.1^\circ})\,(0.85 + j\,1.42)}{0.443 + j\,1.13 + 0.85 + j\,1.42} = 0.70 \underline{/65.3^\circ}$$

and

$$\frac{V_F}{V_f} = \left| 1 + \frac{0.70 \underline{/65.3^\circ}}{14.2 \underline{/42.8^\circ}} \right| = 1.046$$

Using steady-state impedance $\bar{Z}_f = 1.15 \underline{/70.1^\circ}$ ohms, and

$$\frac{V_F}{V_f} = \left| 1 + \frac{1.15 \underline{/70.1^\circ}}{14.2 \underline{/42.8^\circ}} \right| = 1.072$$

Comparison of Measured and Calculated Voltage Ratios for the Complete Series of Tests Using the Simulated Three-Phase Welder Load. Table 6 shows a comparison of measured and calculated values for several "on" and "off" conditions. This comparison shows a good agreement for all tests in which no motor load was connected. In those cases involving motor loads a good agreement in measured and calculated voltage ratios was found for "off" periods which were either very short or were relatively long. For short "off" periods the voltage ratio was calculated using the blocked-rotor motor impedance; for long "off" periods the steady-state motor impedance was used. For intermediate "off" periods the voltage ratios were again numerically between those calculated for short and long "off" periods. The determination of the voltage change for such an "off" period is discussed more fully in Chapter V.

Table 6
Measured and Calculated Voltage Ratios for All of the Experimental Tests Using a Simulated Three-Phase Welder Load

Off Time Cycles	*On Time Cycles*	*Voltage*	*Measured Ratios*		*Calculated Ratios*		
			Without Motors	With Motors	Without Motors	With Motors	
						Using Blocked-Rotor Impedance	Using Steady-State Impedance
1	5½	AB	1.082	1.052	1.079	1.046	1.073
		BC	*	1.050	*	1.046	1.073
		CA	1.075	1.057	1.078	1.046	1.073
2	4½	AB	1.080	1.062	1.078	1.047	1.074
		BC	*	1.070	*	1.047	1.074
		CA	1.079	1.063	1.080	1.046	1.073
3	3½	AB	1.083	1.063	1.083	1.045	1.071
		BC	*	1.069	*	1.046	1.073
		CA	1.083	1.065	1.082	1.045	1.071

* Values not available.

V. METHOD FOR CALCULATING VOLTAGE CHANGES FOR INTERMEDIATE "OFF" PERIODS

The discussion of Chapter III and the data in Chapter IV indicate that the voltage ratios may be calculated accurately for certain "off" periods. If the "off" period is extremely short a value calculated by using the blocked-rotor motor impedance is in close agreement with measured values. Likewise, for long "off" periods the calculated value is in good agreement with the measured value provided the steady-state motor impedance is used in the calculations.

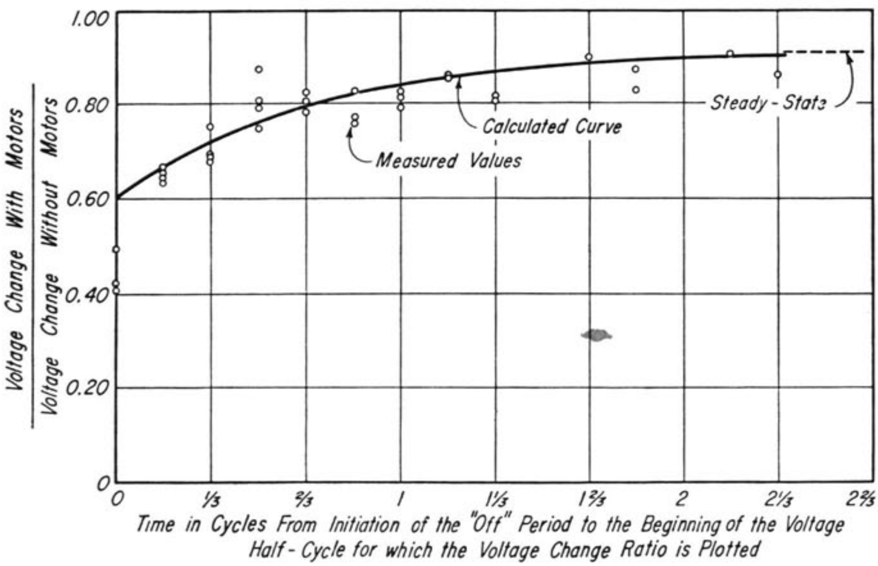


Fig. 13. Variation of Voltage Change During "Off" Period of Simulated Welder in Fig. 10

The above paragraph indicates the need for a method of calculating the voltage change for those cases in which the "off" period is in the range of one to three cycles. In order to give some information concerning calculation methods for such intermediate "off" periods the graph of Fig. 13 was prepared using data from the experimental work discussed in Section 10 of Chapter IV. This graph shows the measured ratio of the voltage *change* occuring with a motor load connected to the system, to the voltage *change* that would exist if there were no motor load connected to the system. Each value of voltage change ratio which is plotted must be associated with a particular half-cycle of one of the system line-to-line voltages. This is true since the voltage change is obtained as the difference between the rms value for a half-cycle of voltage during the "off" period, and the rms voltage for the last half-cycle during the

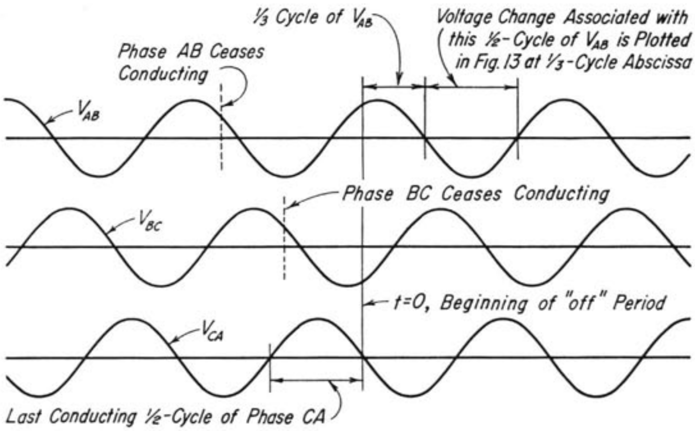


Fig. 14. Graph Used in Determining the Time Coordinates for the Points Plotted in Fig. 13

"on" period. Thus each value of voltage change has associated with it one of the half-cycles of voltage occurring during the "off" period. Therefore, each value of voltage change ratio is plotted in Fig. 13 with an abscissa equal to the time from the beginning of the "off" period to the beginning of the half-cycle of voltage associated with the voltage change ratio. Figure 14 illustrates the manner in which the abscissa is measured for a particular half-cycle of voltage V_{AB}.

For a particular voltage, say V_{AB}, there will be values of voltage change ratio plotted at half-cycle intervals after the initiation of the "off" period. However, the graph of Fig. 13 also shows the voltage change ratios for phases BC and CA. Thus points are plotted every ⅙ of a cycle when all three phases are considered.

The test values which are plotted could not all be expected to fall exactly on any one curve. However, it does appear that an average curve could be drawn so as to come very close to most of the observed values. It also appears that such a curve would have a shape which is approximately exponential.

However, the procedure actually used in plotting the curve shown in Fig. 13 was not that of drawing an average curve. The curve actually shown is exponential in shape and has end points that were calculated by using machine impedances applicable to very short and very long "off" periods. The value of 0.60 at zero time was computed by comparing the voltage change calculated using blocked-rotor motor impedance with the change calculated without considering any motor load. The value of 0.91 at infinite time was calculated in a similar manner by using the steady-state motor impedance. The time constant of this curve was taken as the time constant of the alternating component of current resulting from a sudden change of voltage at the terminals of an induction motor. This time constant can be found by the following expressions.*

$$\text{Time Constant in Cycles} = \frac{\text{Blocked-Rotor Reactance per Phase}}{2\pi\ (\text{Rotor Resistance per Phase})} \tag{17}$$

$$\text{Time Constant in Cycles} = \frac{\text{Percent Blocked-Rotor Reactance}}{2\pi\ (\text{Percent Rotor Resistance})} \tag{18}$$

If the kva base for the induction motor constants is taken as 0.746 times rated hp, the percent rotor resistance of an induction motor can be shown to be very nearly equal to 0.746 times the full-load percent slip. Equation 18 then becomes

$$\text{Time Constant in Cycles} = \frac{(\%\ \text{Blocked-Rotor Impedance})\sin\Theta}{2\pi\ (\%\ \text{Slip at Full Load})\ 0.746} \tag{19}$$

where θ is the angle between voltage and current under starting or blocked-rotor conditions. Using the above mentioned base kva, the percent blocked-rotor impedance can be shown to be equal to 0.746 times the reciprocal of the blocked-rotor kva per hp. Making this substitution for blocked-rotor impedance Eq. 19 becomes

$$\text{Time Constant in Cycles} = \frac{(0.746)\ (\sin\Theta)}{2\pi\ (\%\ \text{Slip at Full Load})\ (\text{Blocked-Rotor kva/hp})\ (0.746)} \tag{20}$$

$$\text{Time Constant in Cycles} = \frac{(\sin\Theta)\ \text{Synchronous RPM}}{2\pi\ (\text{Slip at Full Load in RPM})\ (\text{Blocked-Rotor kva/hp})} \tag{21}$$

Equation 21 may be simplified still further by assuming that $\sin\theta \approx 1$. This assumption is a fairly close approximation for the larger size group

* "Electrical Transmission and Distribution Reference Book," Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pennsylvania, 1944, p. 148.

of general purpose induction motors, since the angle θ approaches 90 deg for such motors. Thus for large motors

$$\text{Time Constant in Cycles} = \frac{\text{Synchronous RPM}}{2\pi\ (\text{Full Load Slip in RPM})\ (\text{Blocked-Rotor kva/hp})} \tag{22}$$

The motor used in the tests of Chapter IV had the following characteristics:

7½-hp, 230-volt, 3-phase
Full-Load Speed = 1145 rpm
Blocked-Rotor Current (½ voltage) = 40.1 amp
Blocked-Rotor Impedance Angle = 59.2°

The time constant of this motor computed by using Eq. 21 is 0.71 cycles. The motor was not large enough to permit the approximation of sin $\theta = 1$. This time constant of 0.71 cycles was used in plotting the curve shown in Fig. 13.

It may be seen from Fig. 13 that the computed curve is in close agreement with the measured values. One would expect such an agreement to exist because the end points of the curve can be computed using blocked-rotor and steady-state motor impedances. Also since the impedance of the motor changes exponentially during a long "off" period from the blocked-rotor value to the steady-state value, it might be expected that the voltage change during the "off" period would have an exponential variation.

The variation just observed of the measured voltage changes for intermediate "off" periods permits the calculation of these changes to be made in the following manner. The voltage change for zero "off" time is computed using the blocked-rotor motor impedance. The voltage change for a long "off" period is computed using the steady-state motor impedance. The motor time constant is computed using either Eq. 21 or 22. Then if

ΔV = Actual voltage change for "off" period of problem
ΔV_{BR} = Voltage change using blocked-rotor impedance
ΔV_{SS} = Voltage change using steady-state impedance
T_M = Motor time constant in cycles
T_O = "Off" time in cycles

the equation showing the relationship between these quantities for an exponential curve like Fig. 13 is

$$\Delta V = \Delta V_{SS} - (\Delta V_{SS} - \Delta V_{BR})\, e^{-\frac{T_O}{T_M}} \tag{23}$$

This equation can be used to calculate an approximate value for the voltage change for any "off" period provided the various quantities are known. However, the value of T_M is not generally known by the utility engineer for the particular motor in question. For this reason values of the exponential factor in Eq. 23 have been computed using average values of motor time constants for various sizes of standard general-purpose motors. These values are sufficiently accurate for those cases in which no motor data other than the hp rating is given. They are listed in Table 7 for several "off" times; in welder terminology, "off" time is frequently referred to as "cool" time. By using a properly selected value of $e^{-\frac{T_O}{T_M}}$ in Eq. 23, the voltage change for the particular "off" period can be found provided ΔV_{SS} and ΔV_{BR} have been calculated.

Table 7

Values for Factor $e^{-\frac{T_O}{T_M}}$ **Given in Eq. 23**

Off Time (Cool Time)	*Range of Induction Motor Horsepower* 5	*7½–15*	*20–75*	*100–200*	*300–500*
No Off Time	1	1	1	1	1
1 Cycle	0.10	0.21	0.44	0.55	0.60
2 Cycles	——	0.05	0.19	0.30	0.36
3 Cycles	——	——	0.08	0.16	0.21
Long Off Time	0	0	0	0	0

In most practical cases it will be found that information concerning the steady-state and blocked-rotor impedances is not available for the calculation of ΔV_{SS} and ΔV_{BR}. In the majority of the problems it will be sufficiently accurate to use the average values given in Table 8, page 37. The steady-state impedance, of course, depends to a great extent upon the load on the motor and, therefore, average values can be given only for a particular load on the motor, say full-load.

VI. OUTLINE OF ANALYTICAL AND GRAPHICAL METHODS FOR CALCULATING THE VOLTAGE CHANGE CAUSED BY THREE-PHASE WELDERS

In order to make the methods given in previous sections more readily available for their application to a problem, a summary of the methods is given in Section 11. This summarizes the complete procedure for the calculation of the voltage change caused by a three-phase welder with any length "off" period.

The procedure summarized in Section 11 may be followed in the solution of a given problem, but the time involved in such a calculation may be quite long. This is particularly true if a number of such problems must be solved. A graphical procedure that can be used in conjunction with established calculation procedures has been developed and is given in Section 12. The method is that of applying a correction for the motor load at P to the voltage change at P calculated without benefit of any motor load.

11. Summary of Voltage Change Calculation Methods

The percent voltage change at the bus supplying the welder, that is, the point F in Fig. 3, can be calculated by the following equation:

$$\text{Percent Voltage Change at } F = (V_F/V_f - 1)\,100$$

where

V_F = voltage at F at the end of the "off" period
V_f = voltage at F at the end of the "on" period

and the voltage ratio V_F/V_f is determined by using Eq. 4. The symbols in Eq. 4 are defined on pages 11-12.

The voltage change at the primary of the welder supply transformer bank is usually more important to the utility engineer than the voltage change at the welder. The percent voltage change at the primary of the transformer bank, which is the point P in Fig. 3, can be calculated by

$$\text{Percent Voltage Change at } P = (V_P/V_p - 1)\,100$$

where

V_P = voltage at P at the end of the "off" period
V_p = voltage at P at the end of the "on" period

and the voltage ratio V_P/V_p is determined by the following:

Eq. 14 when there are motors at P and F
Eq. 15 when there are motors at P only
Eq. 16 when there are motors at F only

The symbols in Eq. 14, 15, and 16 are defined on page 13.

The impedance $\bar{Z}_{MF}{}^*$ which appears in the voltage ratio equations represents the impedance of the motor load at F under steady-state conditions. This impedance is always determined from the impressed voltage, current, and power factor of the motor load. In the case of an induction motor, such an impedance is the same as the impedance determined from the usual equivalent circuit of the motor. For a synchronous motor, an impedance determined from the impressed voltage, current, and power factor is not the impedance shown on the usual equivalent circuit of a synchronous motor, since this consists of an excitation voltage in series with the familiar synchronous impedance.

Long or Very Short "Off" Periods. The impedances $\bar{Z}_{MF}$ and $\bar{Z}_{MP}$ in the voltage ratio equations are the impedances which the respective motor loads present to a change in current at the end of the "off" period. The impedances $\bar{Z}_{MF}$ and $\bar{Z}_{MP}$ can be readily assigned values for very short or long "off" periods. For either $\bar{Z}_{MF}$ or $\bar{Z}_{MP}$ the following impedances apply:

Type of Motor	*Long "Off" Period*	*Very Short "Off" Period*
Induction	Steady-State Impedance (Usual Load Impedance)	Blocked-Rotor Impedance
Synchronous	Synchronous Impedance	Sub-Transient Impedance

The average values of induction and synchronous motor constants given respectively in Tables 8 and 9 can be used to evaluate the various motor impedances in practical cases where data on the actual motors is rarely available. The impedances are given in per unit on the base specified in the table. The time constants for the synchronous motors are given in seconds.

If the motor loads involved are largely synchronous motors, the synchronous motor impedances given in Table 9 and the value of $\bar{Z}_{MF}{}^*$ determined from the steady-state voltage, current, and power factor of the motor load permit the calculation of the voltage change for either a very short or a long "off" period. The range of motor time constants given also permits an estimate of the voltage change for those cases involving "off" times of an intermediate value.

For the usual cases in which the motor loads are predominately induction motors, the impedances given in Table 8 make it possible to calculate the voltage change for very short or long "off" periods. The method of Chapter V may be used to compute the voltage change for intermediate "off" periods. This method is summarized in the following paragraphs of this section.

Intermediate "Off" Periods with Induction Motor Loads. The voltage change for "off" periods intermediate between long and very short "off" periods and for induction motor loads can be determined by Eq. 23. In using this equation, the voltage change ΔV_{SS} for a long "off" period and the voltage change ΔV_{BR} for a very short "off" period are calculated as outlined in the preceding paragraphs of this section. The factor $e^{-\frac{\tau_O}{\tau_M}}$ in Eq. 23 is usually taken from Table 7 since the exact character of the motor loads is rarely known in practical situations.

Table 8
Average Values of Induction Motor Constants
General Purpose, 60-Cycle Motors; 1800, 1200, 900 rpm; 10-500 Hp
(Base kva = Output kw = 0.746 Hp)

Average Full-Load Impedance	$0.8\underline{/27°}$ per unit
Average ¾-Load Impedance	$1.28\underline{/33°}$ per unit
Average Blocked-Rotor Impedance	$0.135\underline{/70°}$ per unit

Table 9
Average Values of Synchronous Motor Constants
General Purpose, 60-Cycle Motors; 1800, 1200, 900 rpm; 25-500 Hp
(Base kva = Rated Armature Kva)

	Range	*Average*
Unsaturated Direct Axis Reactance, X_d	0.60–1.45	1.15 per unit
Transient Reactance, X_d'	0.20–0.50	0.37 per unit
Sub-Transient Reactance, X_d''	0.13–0.35	0.24 per unit
Short Circuit Transient Time Constant, T_d'	0.50–0.65	0.58 sec.
Short Circuit Sub-Transient Time Constant, T_d''	0.01–0.02	0.015 sec.

When there are motors at both P and F in Fig. 3, the effectiveness of each motor load in reducing the voltage change at the point in question must be examined before the factor $e^{-\frac{\tau_O}{\tau_M}}$ can be evaluated. For example, if the motor load at P is much more effective in reducing the voltage change than the motor load at F, the factor $e^{-\frac{\tau_O}{\tau_M}}$ should be determined on the basis of the motor load at P. If the effectiveness of the two motor loads should be about the same, an approximate value for the factor $e^{-\frac{\tau_O}{\tau_M}}$ can be determined by averaging the values of $e^{-\frac{\tau_O}{\tau_M}}$ for the two motor loads.

12. Graphical Solution

The methods which are given in Section 11 may be used to find the voltage change caused by a three-phase welder load. However, the solution of a problem using these methods may require an appreciable amount of time. If it were possible to calculate the voltage change by the usual methods without considering the motor load and then to modify this value to include the effect of the motor loads, a considerable saving in time could be realized.

One system configuration occurs so often that such a correction method has been developed for the computation of the voltage change at point P. This correction includes only the effect of the motor load at point P as shown in Fig. 5. However, this system may also be considered as a simplification of one in which the beneficial effect of the motor loads at point F is neglected.

This correction method makes use of a factor K which is shown in the chart of Fig. 15. The voltage change at point P is first calculated by the usual method without considering any motor load. This voltage change is then multiplied by the correction factor K to obtain the actual voltage change at P (effect of the motors taken into account). The values of K on the chart of Fig. 15 were determined from the results of a number of calculations using the equations referred to in Section 11.

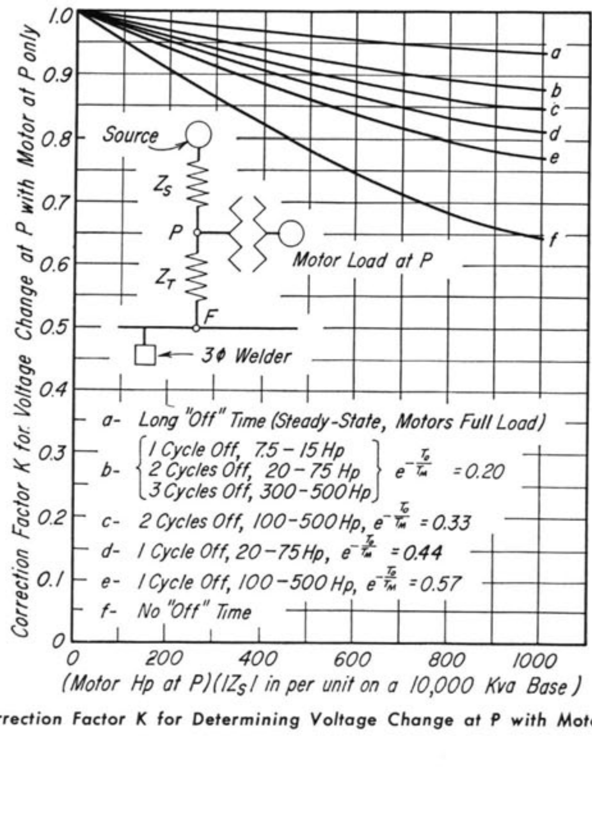


Fig. 15. Correction Factor K for Determining Voltage Change at P with Motors at P Only

Several curves for finding K are given on the chart. These curves show the variation of K with the welder "off" time and the size of the motors at P. The several curves are necessary to take into account the variation of voltage change with "off" time and motor time constant as indicated in Eq. 23.

The independent variable on the chart is the product, $(|Z_S|)$ (Motor hp at P), where $|Z_S|$ is given in per unit on a 10,000 kva base. The motor load at P is taken entirely as induction motors with transformers at 4 percent impedance on a base kva equal to 0.746 times the motor hp at P.

When the effect of motor loads at F must be considered, the equations referred to in Section 11 must still be used. The expansion of the chart to include several other variables was not considered justified because of the relatively few times that the motor load at F is large enough to be important.

13. Numerical Example

Problem

Find the voltage change at P in Fig. 5 due to a three-phase welder at F. Solve the problem by the method outlined in Section 11 and also by the method outlined in Section 12.

Data as Given

System impedance to point $P = \overline{Z}_S = 0.15 \underline{/65°}$ p.u. on a 10,000 kva base

Transformer impedance $= \overline{Z}_T = 0.05 \underline{/90°}$ p.u. on a 1000 kva base

Welder load (equivalent sinusoidal) = 1000 kva at 85% power factor lag

Welder "off" time = 1 cycle

Motor load at F: none

Induction motor load at P: 3000 hp ranging from 7½-15 hp and operating at full load; 3000 hp ranging from 20-75 hp and operating at full load

Motor Impedances

The impedances for the total 6000 hp induction motor load at P can be found by using Table 8. The following motor impedances were determined assuming negligible impedance for the transformers supplying the motor load at P:

$\overline{Z}_{MP} = 0.8 \underline{/27°}$ p.u. Steady-state value on a 4470 (6000×.746) kva base

$\overline{Z}_{MP} = 0.135 \underline{/70°}$ p.u. Blocked-rotor value on a 4470 kva base

Impedances in Per Unit on a 10,000 kva Base

The following impedances have been converted to a common 10,000 kva base:

$\bar{Z}_S = 0.15 \angle 65^\circ$ p.u.

$\bar{Z}_T = 0.50 \angle 90^\circ$ p.u.

$\bar{Z}_W = 10 \angle 31.7^\circ$ p.u.

$\bar{Z}_{MP} = 1.79 \angle 27^\circ$ p.u. Steady-state value.

$\bar{Z}_{MP} = 0.302 \angle 70^\circ$ p.u. Blocked-rotor value.

Voltage Change Determined by Method of Section 11

Using Eq. 15 and the steady-state value for the motor impedance $\bar{Z}_{MP}$

$$\begin{aligned} \frac{V_P}{V_p} &= \left| 1 + \frac{\bar{Z}_{MP}\bar{Z}_S}{(\bar{Z}_W + \bar{Z}_T)(\bar{Z}_{MP} + \bar{Z}_S)} \right| \\ &= \left| 1 + \frac{(1.79 \angle 27^\circ)(0.15 \angle 65^\circ)}{(10 \angle 31.7^\circ + 0.5 \angle 90^\circ)(1.79 \angle 27^\circ + 0.15 \angle 65^\circ)} \right| \\ &= 1.0121 \end{aligned}$$

Therefore the voltage change $\Delta V_{SS} = 1.21\%$.

Using Eq. 15 and the blocked-rotor value for the motor impedance $\bar{Z}_{MP}$

$$\frac{V_P}{V_p} = \left| 1 + \frac{(0.302 \angle 70^\circ)(0.15 \angle 65^\circ)}{(10 \angle 31.7^\circ + 0.5 \angle 90^\circ)(0.302 \angle 70^\circ + 0.15 \angle 65^\circ)} \right| = 1.0083$$

or the voltage change $\Delta V_{BR} = 0.83\%$.

From Table 7 for a one cycle "off" time,

$$e^{-\frac{T_O}{T_M}} = 0.21 \text{ for } 7\tfrac{1}{2}\text{-15 hp range}$$

and

$$e^{-\frac{T_O}{T_M}} = 0.44 \text{ for 20-75 hp range}$$

Since the motor load at P consists of 3000 hp in each of the above ranges, an average value of $e^{-\frac{T}{T_M}} = 0.32$ can be used.

The actual voltage change ΔV can now be determined by Eq. 23:

$$\begin{aligned} \Delta V &= \Delta V_{SS} - (\Delta V_{SS} - \Delta V_{BR})\, e^{-\frac{T_O}{T_M}} \\ &= 1.21 - (1.21 - 0.83)(0.32) \\ &= 1.09\% \end{aligned}$$

The voltage change at point P in Fig. 5 due to the three-phase welder at point F is thus found to be 1.09% using the method of Section 11.

Voltage Change Determined by Method of Section 12

The voltage change at P with no motors connected at P can be determined by using Eq. 15 with $\bar{Z}_{MP} = \infty$. Making this substitution,

$$\frac{V_P}{V_p} = \left| 1 + \frac{\bar{Z}_S}{\bar{Z}_W + \bar{Z}_T} \right| = \left| 1 + \frac{0.15 \,\underline{/65^\circ}}{(10 \,\underline{/31.7^\circ} + 0.5 \,\underline{/90^\circ})} \right| = 1.0126$$

or the voltage change with no motors is 1.26%.

In order to find the value of K by means of the chart of Fig. 15, it is necessary to calculate the product (Motor hp at P) ($|Z_S|$ in per unit on a 10,000 kva base) which is the abscissa on the chart. This product is 6000 x 0.15 = 900. The value of K is determined by using 900 as the abscissa and selecting an ordinate midway between curves b and d. (This ordinate is selected since there are 3000 hp of motors in the 7½-15 hp range and 3000 hp of motors in the 20-75 hp range.) This procedure gives a value of $K = 0.855$.

Following the method of Section 12, the actual voltage change at point P in Fig. 5 caused by the three-phase welder at F is equal to (0.855) (1.26) = 1.08%.

VII. SUMMARY

The following conclusions can be made as a result of the preceding investigation of the effect of polyphase motors on the voltage change caused by a three-phase welding load which is connected to a three-phase supply system.

a. The effect of the motors is to reduce the voltage change caused by the welder. In a practical case this can be appreciable when the "off" period between secondary current pulses is sufficiently short and, therefore, the motor effect should be included in an accurate analysis of the voltage changes caused by the load.

b. The voltage change with the effect of the motor loads included can be calculated by the methods and equations which have been presented. These methods permit a *direct* calculation for welding loads having very short "off" periods or for those whose "off" period is long enough for steady-state conditions to be reached during the "off" period. The methods of Chapter V permit an approximation of the voltage change for welder loads whose "off" period is between the limits mentioned above.

c. In practical cases it is found that the exact character of the motor loads adjacent to the three-phase welder cannot usually be determined. It is also found that these loads are largely composed of induction motors. Because of this, most practical cases can be handled with sufficient accuracy by using the average full-load, ¾-load, and blocked-rotor impedances given in Table **8** on page **37**. For the small number of cases involving a large amount of synchronous motor load either the actual motor impedances or the average values given in Table **9** on page **37** would have to be used.

d. The chart given in Fig. **15** provides a correction factor K by which the voltage change computed without effect of the motor loads may be modified to include this effect. The use of this method can result in a considerable saving in time since the calculation of the voltage change by using the equations of Chapter II may be a long process. The use of the chart, however, is limited to the voltage change at P and to the circuit configuration which is specified on the chart.

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